In selecting the appropriate design approach for structural concrete, it is useful to classify portions of the structure as either B – (Beam or Bernoulli) Regions or D – (Disturbed or Discontinuity) Regions. B – Regions are parts of a structure in which Bernoulli's hypothesis of straight-line strain profiles applies. D–Regions, on the other hand, are parts of a structure with a complex variation in strain. D–Regions include portions near abrupt changes in geometry (geometrical discontinuities) or concentrated forces (statical discontinuities). Based on St. Venant's principle, the extent of a D–Region spans about one section depth of the region on either side of the discontinuity.

Figure 1 and Figure 2 show examples of the division between B – Regions and D–Regions in building and bridge structures, respectively. In the figures, the unshaded area with a notation B indicates B–Region, and the shaded area with a notation D is used to indicate D–Region. The notations h_1, h_2, h_3, ... are used to denote the depth of structural members. The notations b_1 and b_2 denote the flange width of structural members.

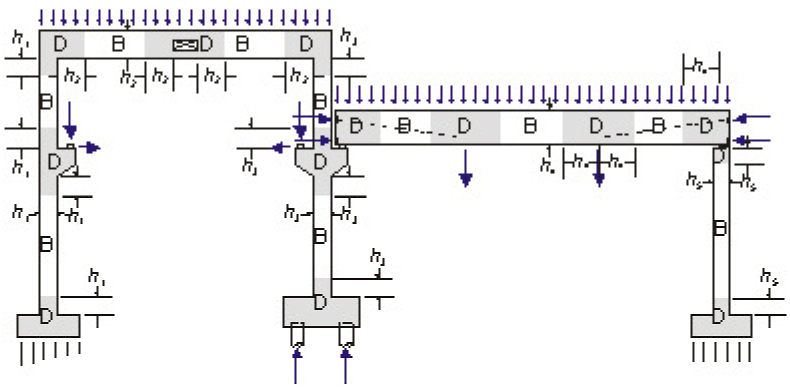

Figure 1 Example of D-Regions in a Common Building Structure

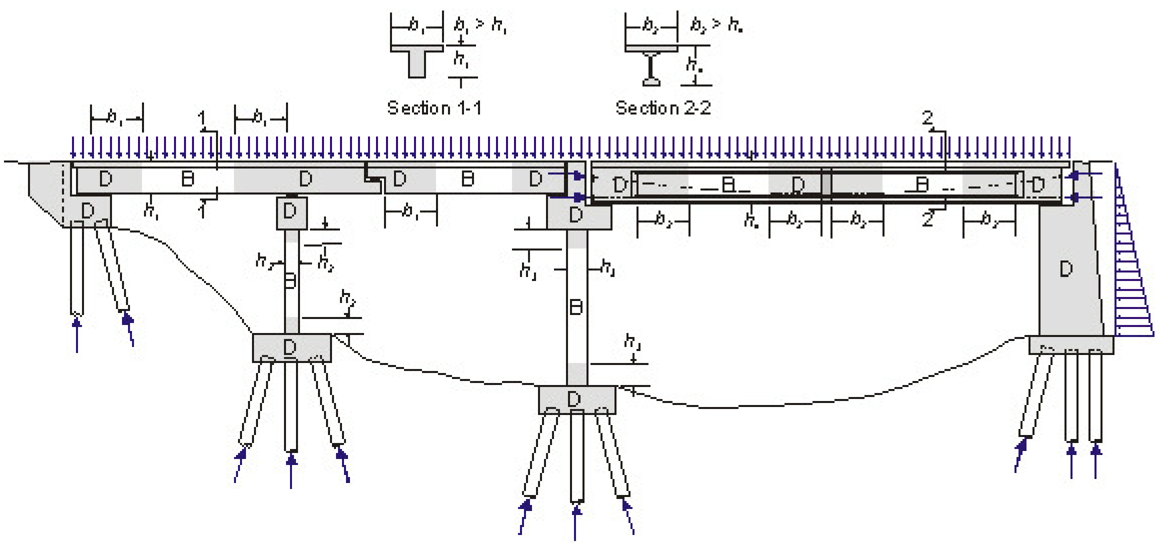

Figure 2 Example of D-Regions in a Common Bridge Structure

Most design practices for B-Regions are based on a model for behavior. As examples, design for flexure is based on conventional beam theory while the design for shear is based on the well-known parallel chord truss analogy. By contrast, the most familiar types of D-Regions, such as deep beams, corbels, beam-column joints, and pile caps, are currently still designed by empirical approaches or by using common detailing practices. For most other types of D-Regions, code provisions provide little guidance to designers. The Strut-and-Tie Method (STM) [1-3] is emerging as a code-worthy methodology for the design of all types of D-Regions in structural concrete.

It is worth noting that although the STM is equally applicable to both B- and D-Region problems, it is not practical to apply the method to B-Region problems. The conventional beam theory for flexure and parallel chord truss analogy for shear are recommended for those designs.

The STM is based on the lower-bound theory of limit analysis. In the STM, the complex flow of internal forces in the D-Region under consideration is idealized as a truss carrying the imposed loading through the region to its supports. This truss is called *strut-and-tie model* and is a statically admissible stress field in lower-bound (static) solutions. Like a real truss, a strut-and-tie model consists of *struts* and *ties* interconnected at *nodes* (also referred to as *nodal zones* or *nodal regions*). A selection of strut-and-tie models for a few typical 2-D D-Regions is illustrated in Figure 3. As shown in the figure, struts are usually symbolized using broken lines, and ties are usually denoted using solid lines.

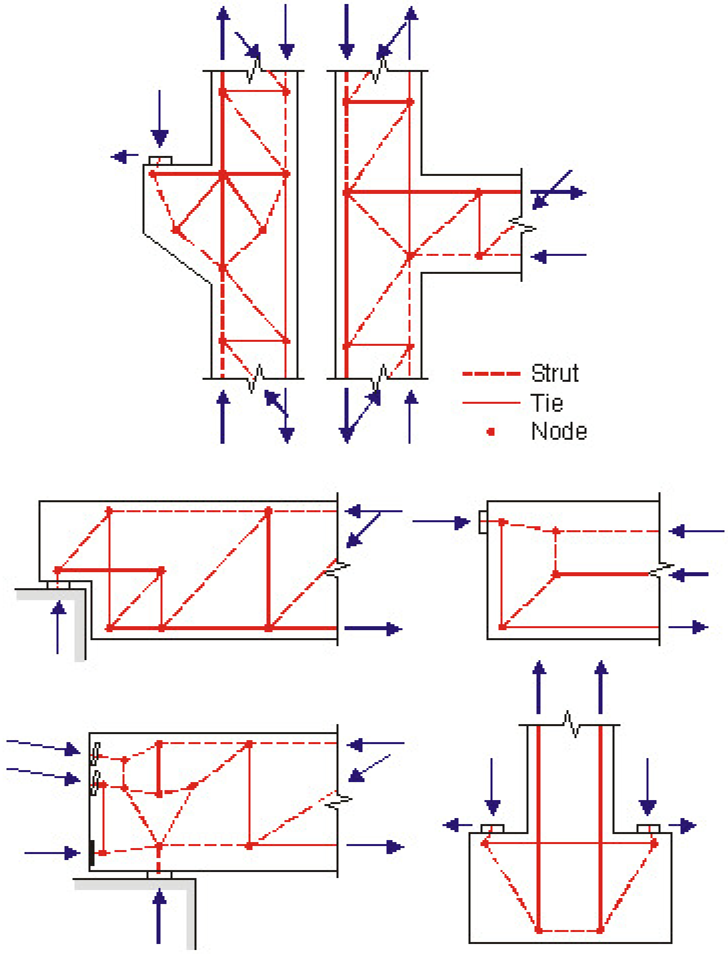

Figure 3 Examples of Strut-and-Tie Models for Common Structural Concrete Members

Strut-and-Tie Model Components

Struts are the compression members of a strut-and-tie model and represent concrete stress fields whose principal compressive stresses are predominantly along the centerline of the strut. The idealized shape of concrete stress field surrounding a strut in a plane (2-D) member, however, can be prismatic (Figure 4(a)), bottle-shaped (Figure 4(b)), or fan-shaped (Figure 4(c)) [3]. Struts can be strengthened by steel reinforcement, and if so, they are termed reinforced struts.

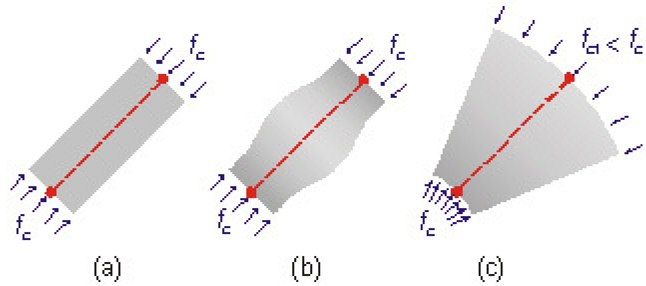

Figure 4 Basic Type of Struts in a 2-D Member: (a) Prismatic (b) Bottle-Shaped (c) Fan-Shaped

Ties are the tension members of a strut-and-tie model. Ties mostly represent reinforcing steel, but they can occasionally represent prestressing steel or concrete stress fields with principal tension predominant in the tie direction.

Nodes are analogous to joints in a truss and are where forces are transferred between struts and ties. As a result, these regions are subject to a multidirectional state of stress. Nodes are classified by the types of forces being connected. Figure 5 shows basic types of nodes in a 2-D member; in the figure, *C* is used to denote compression and *T* is used to denote tension.

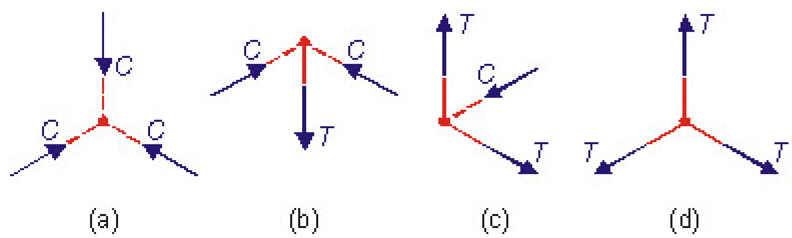

Figure 5 Basic Type of Nodes: (a) CCC (b) CCT (c) CTT (d) TTT

Uniqueness of Strut-and-Tie Models

As a statically admissible stress field, a strut-and-tie model has to be in equilibrium externally with the applied loading and reactions (the boundary forces) and internally at each Node. In addition, reinforcing or prestressing steel is selected to serve as the ties, the effective width of each strut is selected, and the shape of each nodal zone is constructed such that the strength is sufficient. Therefore, only equilibrium and yield criterion need to be fulfilled for an admissible strut-and-tie model. The third requirement in solid mechanics framework, namely the strain compatibility, is not considered.

As a result of these relaxed requirements, there is no unique strut-and-tie model for a given problem. In other words, more than one admissible strut-and-tie model may be developed for each load case as long as the selected truss is in equilibrium with the boundary forces and the stresses in the struts, ties, and nodes are within the acceptable limits. The lower-bound theorem guarantees that the capacity obtained from all statically admissible stress fields is lower than or equal to the actual collapse load. However, as a result of limited ductility in the structural concrete, there are only a small number of viable solutions for each design region. Figure 6 illustrates an example in which one solution is preferable to another. Due to the point load at the tip of the cantilever portion, the upper part of the beam is likely to develop horizontal tensile stresses along the beam. Therefore, the model with the upper horizontal tie (Figure 6(a)) is preferable to that shown in Figure 6(b). The latter only effectively resists the tension in the upper region near the middle support.

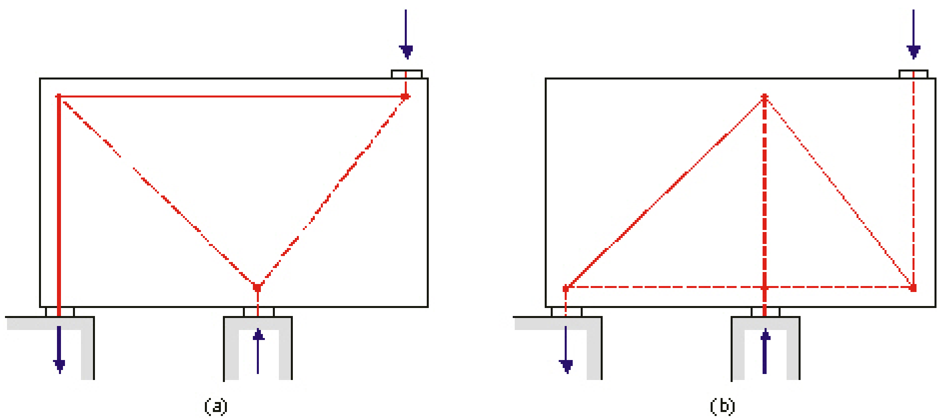

Figure 6 Two statically admissible strut-and-tie models for a cantilevered deep beam under vertical loading: (a) Workable truss (b) Less favorable truss due to excessive ductility demands

The design process using STM involves five major steps described below. These steps are illustrated in Figure 7 using the design example of a dapped-ended beam.

1. Define the boundaries of the D-Region and determine the boundary forces (the ultimate design forces) from the imposed local and sectional forces.
2. Sketch the truss, determine the equivalent boundary forces, and solve for the truss member forces.
3. Select reinforcing or prestressing steel to provide the necessary tie capacity and ensure that this reinforcement is properly anchored in the nodes.
4. Evaluate the dimensions of the struts and nodes such that the capacity of all struts and nodes is sufficient to carry the truss member forces.
5. Provide distributed reinforcement to ensure ductile behavior of the D-Region.

Since equilibrium of the truss with the boundary forces must be satisfied (step 2) and stresses everywhere must be below the limits (step 3 and 4), one can see that the STM is a lower-bound (static or equilibrium) method of limit analysis.

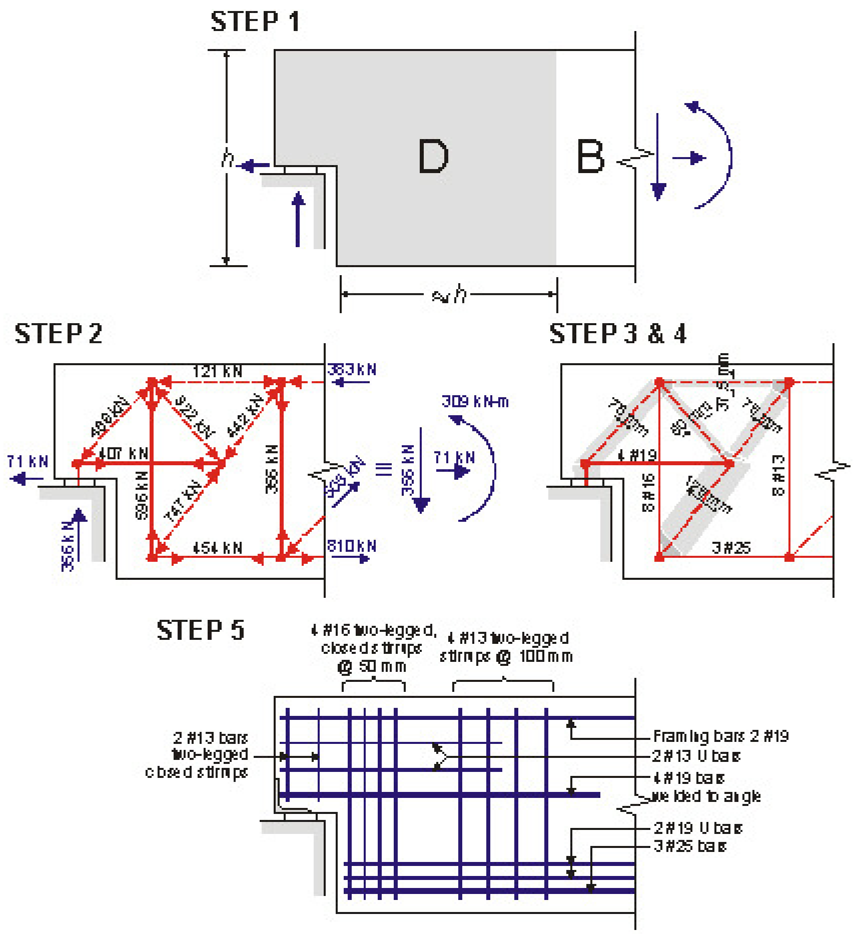

Figure 7 The Major Steps in STM Design Process

STM design provisions consist of rules for defining the dimensions and ultimate stress limits of struts and nodes as well as the requirements for the distribution and anchorage of reinforcement. Guidelines [5, 6] for design by the STM have been developed for European practice. Provisions for the STM have been incorporated in the Canadian Concrete Design Code [7, 8] since 1984 and in the AASHTO LRFD [9, 10] code since 1994. Another specific set of provisions has been developed to be included as an alternative design procedure in the 2002 ACI code [11].

Table 1 and Table 2 show examples of stress limits and strength reduction factors defined in ACI Code and AASHTO LRFD Bridge Design Specifications, respectively. As shown in the tables, there are substantial differences in the rules used in these provisions and guidelines because of uncertainties associated with defining the characteristics of an idealized truss within a continuum of structural concrete.

Table 1 Stress Limits and Strength Reduction Factors According to ACI 318-02 Appendix A [11]

Stress Limits, f_{cu}
Struts: $f_{cu} = 0.85\beta_s f_c'$
where: $\beta_s = 1.00$ for prismatic struts in uncracked compression zones $\beta_s = 0.40$ for struts in tension members $\beta_s = 0.75$ struts may be bottle shaped and crack control reinforcement is included $\beta_s = 0.60$ struts may be bottle shaped and crack control reinforcement is not included $\beta_s = 0.60$ for all other cases $f_c' =$ specified concrete compressive strength
Note: Crack control reinforcement requirement is $\sum \rho_{vi} \sin \gamma_i \geq 0.003$, where ρ_{vi} = steel ratio of the *i*-th layer of reinforcement crossing the strut under review, and γ_i = angle between the axis of the strut and the bars.
Nodes: $f_{cu} = 0.85\beta_n f_c'$
where: $\beta_n = 1.00$ when nodes are bounded by struts and/or bearing areas $\beta_n = 0.80$ when nodes anchor only one tie $\beta_n = 0.60$ when nodes anchor more than one tie
Strength Reduction Factors, ϕ
$\phi = 0.75$ for struts, ties, and nodes

Table 2 Stress Limits and Strength Reduction Factors According to AASHTO LRFD Bridge Design Specifications 2nd Edition [10]

Stress Limits, f_{cu}

Struts: $f_{cu} = \dfrac{f_c'}{0.8 + 170\varepsilon_1} \le 0.85 f_c'$

where: $\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)\cot^2\theta_s$

θ_s = smallest angle between the strut under review and the adjoining ties

ε_s = average tensile strain in the tie direction

f_c' = specified concrete compressive strength

Note: The stress limit assumes a minimum distributed reinforcement of 0.003 in each direction is provided.

Nodes: $f_{cu} = \nu f_c'$

where: ν = 0.85 when nodes are bounded by struts and/or bearing areas

ν = 0.75 when nodes anchor only one tie

ν = 0.65 when nodes anchor more than one tie

Resistance Factors, ϕ

ϕ = 0.7 for struts and nodes

ϕ = 0.9 for ties

Introduction and Project Significance

1.1 B- (Beam) and D- (Discontinuity) Regions

B- (Beam) Regions are those parts of the structure in which there is a linear variation in strain over the depth of the member, while D- (Discontinuity) Regions are those parts of a structure in which there is a complex variation in strain. Based on St. Venant's principle, D-Regions are those parts of a structure within a distance equal to the depth of the member from a concentrated force (load or reaction point), change in section depth, an opening, or another discontinuity. As Figure 1 illustrates, a large portion of even common structures are D-Regions.

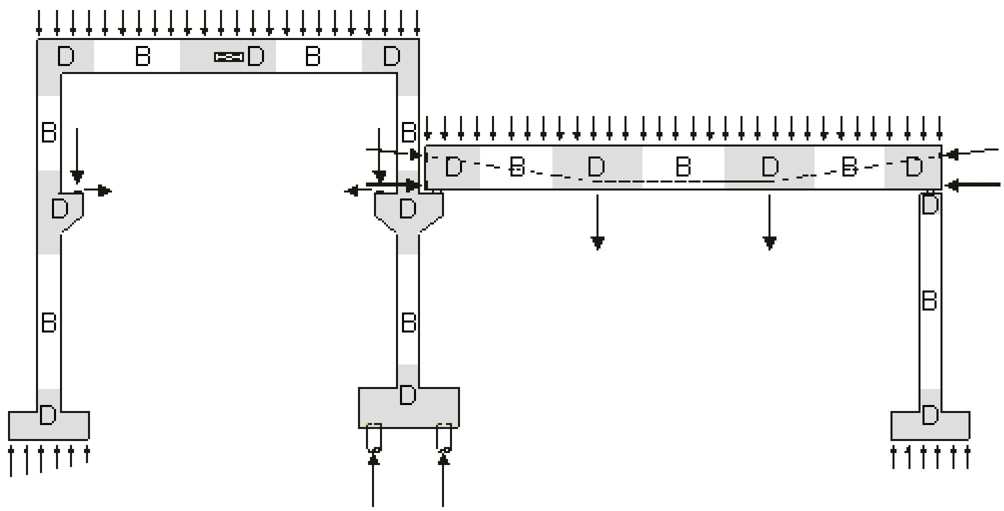

Figure 1 Example of D-Regions in Common Structures

1.2 Shortcoming of Common Design Practice for D-Regions

Empirical code provisions and/or unstandardized detailing practice are used for designing the most familiar types of D-Regions, such as deep beams, corbels, joints, and pile caps. These procedures are unacceptably inexact, which leads to deficiencies or inefficiencies in the design of these commonly occurring and often critical parts of structures. To illustrate this point, Figure 2 demonstrates the inability of the ACI 318-99 [1] provisions to reasonably well estimate the shear strength of deep beams [2, 3]. If provisions were adequate and efficient, the V_{test}/V_{ACI} ratio for the vast majority of the results would lie between about 1 and 1.25. Provisions and practices for the design of other types of D-Regions are unlikely to be better and are probably less accurate than those for the comparatively simple and heavily-tested deep beam. Another shortcoming of current design provisions is that engineers are provided with little to no guidance for the design of less common or unique D-Regions. Due to the inadequacies in common practice, coupled with the unlimited variety of D-Region shapes and loading conditions, it is not surprising that most structural problems occur in D-Regions.

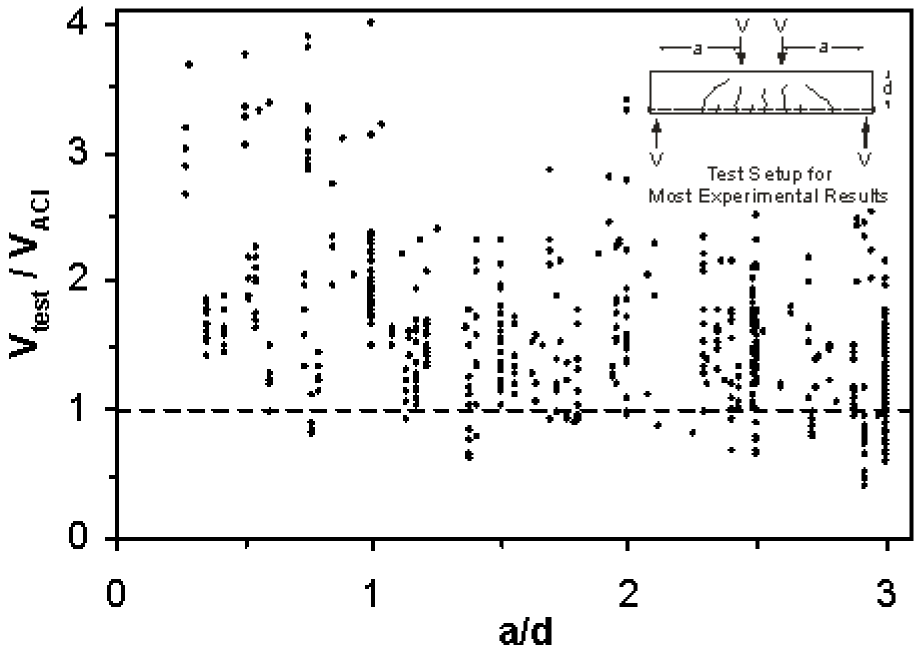

Figure 2 Shortcomings of Existing Provisions

1.3 The Strut-and-Tie Method (STM) for the Design of D-Regions

An emerging methodology for the design of all types of D-Regions is to envision and design an internal truss, consisting of concrete compressive struts and steel tension ties that are interconnected at nodes, to support the imposed loading through to the boundaries of the discontinuity region. This design methodology is called the Strut-and-Tie Method (STM) [4-9]. The design process involves the steps described below. In Figure 3, these steps are illustrated using a variety of D-Region designs examples including a corbel, a corner joint, a dapped-ended beam, and a deep beam.

(i) Define the boundaries of the D-Region and determine the imposed local and sectional forces.

(ii) Sketch the internal supporting truss, determine equivalent loadings, and solve for truss member forces.

(iii) Select reinforcing or prestressing steel to provide the necessary tie capacity and ensure that this reinforcement is properly anchored in the nodal zone (joint of the truss).

(iv) Evaluate the dimensions of the struts and nodes, such that the capacity of these components (struts and nodes) is sufficient to carry the design forces values.

(v) Provide distributed reinforcement to ensure ductile behavior of the D-Region.

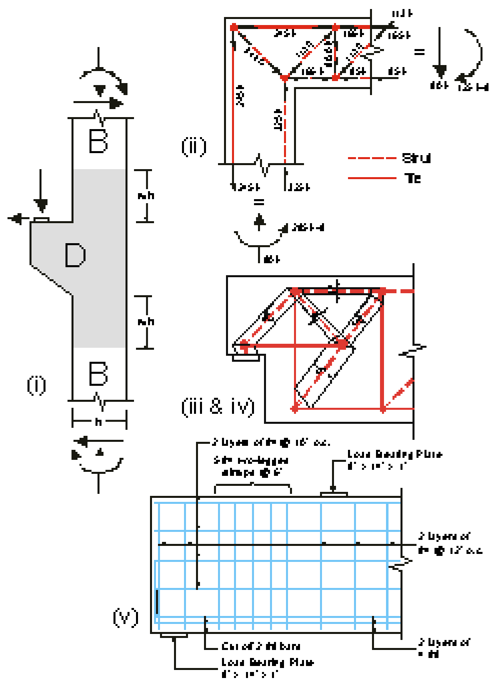

Figure 3 Strut-and Tie Models and Steps in Design

The STM is based on the lower bound theory of plasticity. Therefore, the actual capacity of the structure is considered to be equal to or greater than that of the idealized truss. This suggests that if Truss A (Cut-Away Truss shown in Figure 4) can support a load of PA, then the capacity PB of Deep Beam B (equivalent to Truss A + three concrete fills) is at least equal to PA. This statement is almost true. In the “filled-in” structure, the forces may spread out along the length of the strut resulting the strut failing by splitting at a lower load than it would have failed by crushing at had the stress trajectories been parallel. Such effects can, however, be easily accounted for in provisions by reducing ultimate stress limit values.

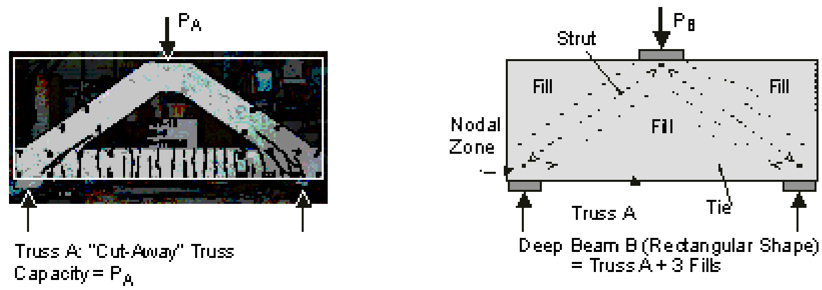

Figure 4 Illustration of “Cut-Away” and “Filled-In” Truss

STM design provisions consist of rules for defining the maximum dimensions and ultimate stress limit capacities of struts and nodes, as well as reinforcement anchorage and distribution requirements. Existing and proposed code provisions differ substantially due to uncertainties in what these rules should be. This situation is created by a lack of sufficient and detailed experimental research. Guidelines [10-11] for design by the STM have been developed for European practice. A version of the STM was incorporated in the Canadian Concrete Design Code [12] in 1984 and in the AASHTO LRFD [13] code in 1994. Another specific set of provisions has been developed to include as an alternative design procedure in the 2002 ACI code. These provisions were submitted to the full ACI 318 committee as CE49 [14], and at the time of submission of this proposal, were under revision.

1.4 Complications and Barriers to Design by the STM

While the STM is a conceptually simple design tool, there are numerous uncertainties and complications that can encumber the five-step design procedure. A few of these are briefly described below:

Strut and Node Capacity: The ultimate stress at failure in struts and nodal zones is influenced by several factors including shape, state of strain/cracking, and the level of confinement. The influence of these factors is poorly understood and this leads to uncertainties in the design method. Additionally, designers are not able to take advantage of factors that they believe would increase capacity or improve behavior.

Geometry of Struts and Nodal Zones: It is unclear how to define the effective dimensions of struts and nodal zones. This is particularly difficult for configurations in which more than three members intersect. An example of such a complex strut-and-tie model is illustrated in Figure 5. Since the capacity of the struts and nodes are directly proportional to their effective widths, this creates uncertainties in the design process.

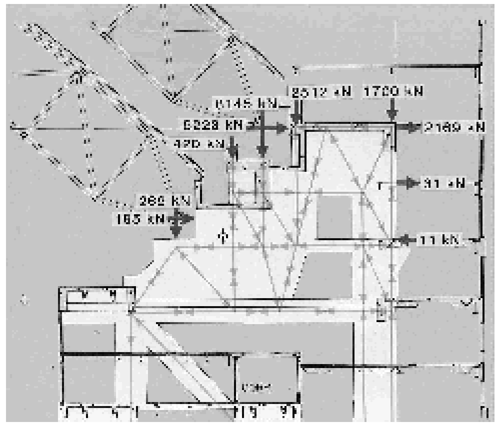

Figure 5 Radial Walls of Skydome, Toronto: Designed using the STM

Anchorage of Tie Reinforcement: In the cut-away truss, the transfer of forces between members and the anchorage of tension ties occurs entirely in the nodal zone. In the full structure ("filled-in" truss), this force transfer is more broadly distributed. There are uncertainties about anchorage requirements, the need to distribute reinforcement throughout the nodal region, and the factors that influence these requirements.

Truss Geometry and Dimensions: The initially selected geometry of the truss, including strut and nodal zone dimensions, must often be adjusted in order to satisfy stress limit criteria, to investigate other configurations, and to optimize the design. This can make hand-solutions prohibitively time consuming, particularly for the design of complex structures for which there is the need to consider multiple load cases.

Statically Indeterminate Trusses: The non-linear axial stiffness characteristics of struts and ties are poorly understood. Consequently, the designer has little guidance for determining the distributions of loads in statically indeterminate strut-and-tie (truss) models.